Congruence Closure with Free Variables: A quantifier-instantiation framework for SMT

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SRI Formal Topics Seminar

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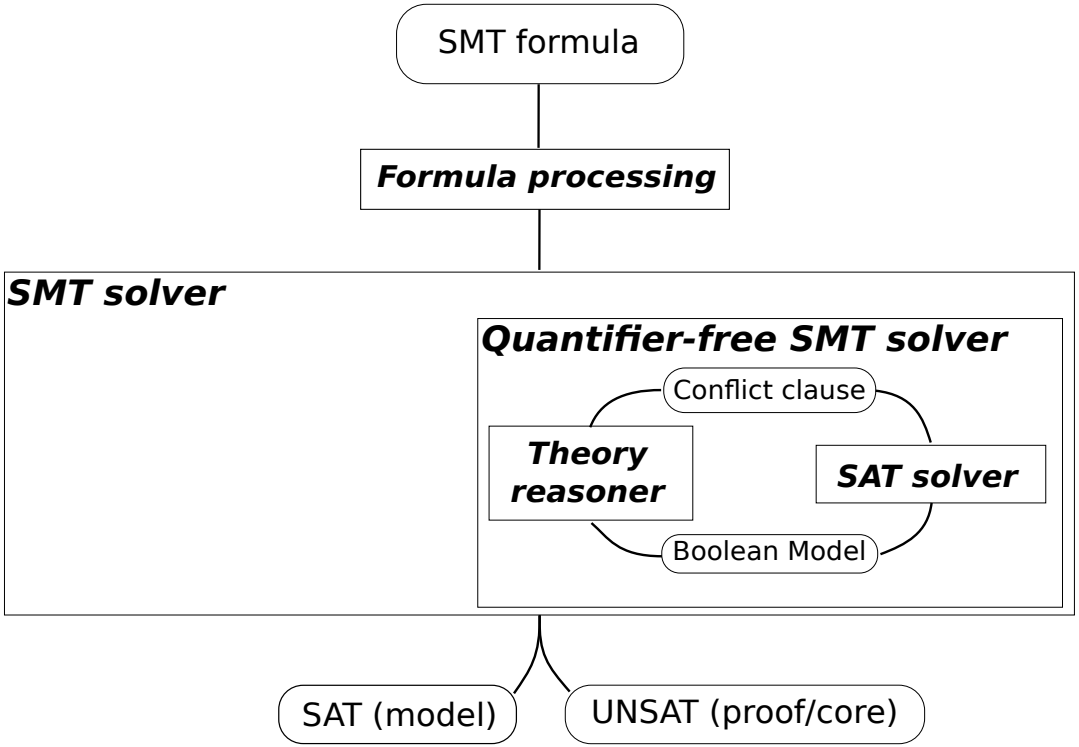
Overview

- Instantiation in SMT
- Congruence closure with free variables
  - E-ground (dis)unification
  - Casting instantiations techniques
  - Decision procedure
  - Implementation
  - Evaluation
- Issues and extensions
  - WIP towards HOSMT

CDCL(T) architecture



Quantifier-free solver enumerates models E

- E is a set of ground literals $\{a \leq b,\ b \leq a + x,\ x \simeq 0,\ f(a) \not\simeq f(b)\}$

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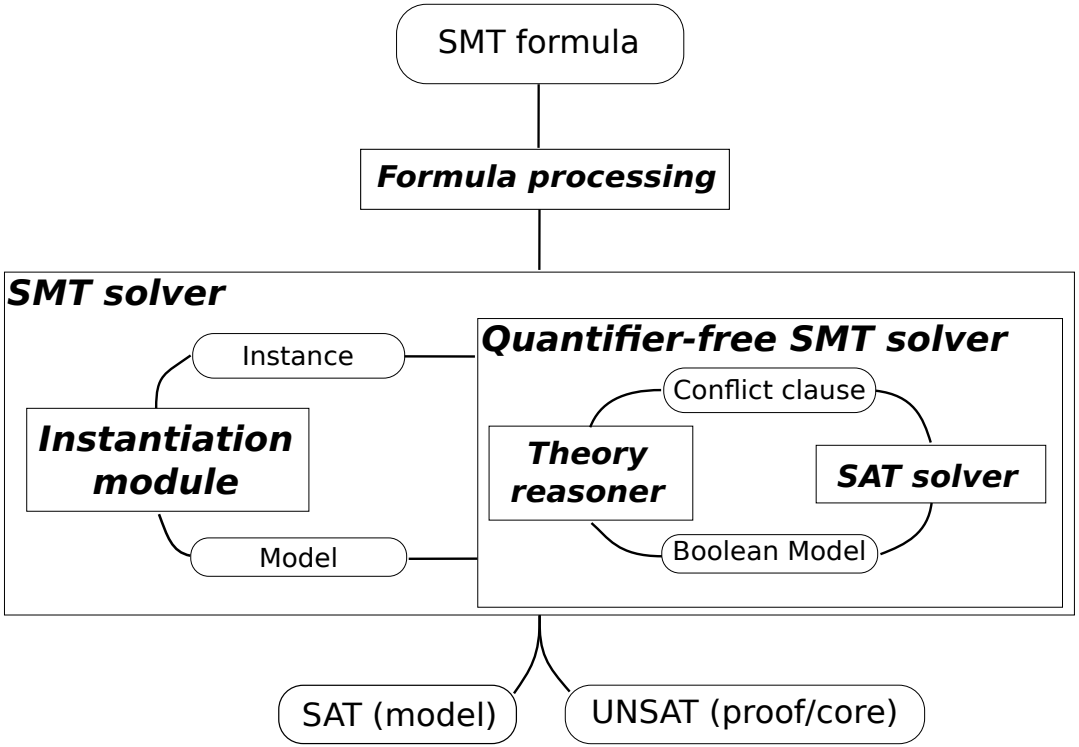

Quantifier-free solver enumerates models $E \cup \mathcal{Q}$

- E is a set of ground literals $\{a \leq b,\ b \leq a + x,\ x \simeq 0,\ f(a) \not\simeq f(b)\}$
- $\mathcal{Q}$ is a set of quantified clauses $\{\forall xyz.\ f(x) \not\simeq f(z) \vee g(y) \simeq h(z)\}$

Instantiation module generates instances of $\mathcal{Q}$ $f(a) \not\simeq f(b) \vee g(a) \simeq h(b)$

Instantiation techniques

- ▷ Trigger-based [DNS05; MB07]
- ▷ Conflict-based [RTM14; BFR17]
- ▷ Model-based [GM09; RTG+13]

- ⊕ General: ∀+EUF+...
- ⊖ Finding instantiations is hard

- ▷ Enumerative [RBF18]
- ⊕ Easy to implement
- ⊕ Reliable last resort

- ▷ QE-based [Mon10; Bjø10; RDK+15; BJ15]
- ⊕ Decision procedures available
- ⊖ Pure fragments

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CCFV is a unifying framework for trigger-, conflict-, and model-based instantiation

A bit of history

Dec. 2013: "Make veriT great on quantifiers. Probably try superposition."

After much suffering acting like a resolution prover...

"A resolution prover is like a prolific but not very well organized mathematician filling notebooks with trivial deductions, with no overall sense of where he is going. Once in a while he stumbles on something interesting." - David A. Plaisted [Pla15]

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Conflict-based instantiation [RTM14]

- ▷ Given theory T, a model $E \cup \mathcal{Q}$, for some $\forall \bar{x}.\ \psi \in \mathcal{Q}$ find σ s.t. $E \wedge \psi\sigma \models_T \bot$
- ▷ Add instance $\forall \bar{x}.\ \psi \to \psi\sigma$ to quantifier-free solver

Finding conflicting instances requires deriving σ s.t. $E \models_T \neg\psi\sigma$

- ⊕ Goal-oriented instantiation technique
- ⊕ Efficient
- ⊖ Specialized solution
- ⊖ Incomplete matching

Let's look deeper into the problem (with $T = \mathrm{EUF}$)

$$E \models \neg\psi\sigma, \text{ for some } \forall\bar{x}.\ \psi \in \mathcal{Q}$$

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E-ground (dis)unification

Given conjunctive sets of equality literals E and L, with E ground, finding a substitution σ s.t. $E \models L\sigma$

- Solution space can be restricted into ground terms from $E \cup L$

- NP-complete
  - NP: solutions can checked in polynomial time
  - NP-hard: reduction of 3-SAT into the entailment

- Variant of classic (non-simultaneous) rigid E-unification

$$s_1\sigma \simeq t_1\sigma, \ldots, s_n\sigma \simeq t_n\sigma \models u\sigma \simeq v\sigma$$

Casting instantiation techniques: Trigger-based

$$E \models (u_1 \simeq y_1 \wedge \cdots \wedge u_m \simeq y_m)\,\sigma$$

where $\{u_1, \ldots, u_m\}$ is a trigger for $\forall \bar{x}.\, \psi \in \mathcal{Q}$ and each $y_i\sigma \in \mathbf{T}(E)$

- Consider
  - $E = \{f(a) \simeq g(b),\ h(a) \simeq b,\ f(a) \simeq f(c)\}$
  - $\mathcal{Q} = \{\forall x.\, f(x) \not\simeq g(h(x))\}$, Trigger=$\{f(x)\}$
- Solving $E \models (f(x) \simeq y)\sigma$ yields
  - $\sigma_1 = \{y \mapsto f(a),\ x \mapsto a\}$
  - $\sigma_2 = \{y \mapsto f(c),\ x \mapsto c\}$
- The instantiation lemmas are:
  - $\forall x.\, f(x) \not\simeq g(h(x)) \rightarrow f(a) \not\simeq g(h(a))$
  - $\forall x.\, f(x) \not\simeq g(h(x)) \rightarrow f(c) \not\simeq g(h(c))$

Casting instantiation techniques: Conflict-based

$$E \models \neg\psi\sigma, \text{ for some } \forall\bar{x}.\, \psi \in \mathcal{Q}$$

- Consider
  - $E = \{f(a) \simeq g(b),\ h(a) \simeq b,\ f(a) \simeq f(c)\}$
  - $\mathcal{Q} = \{\forall x.\ f(x) \not\simeq g(h(x))\}$
- Solving $E \models (f(x) \simeq g(h(x)))\sigma$ yields
  - $\sigma = \{x \mapsto a\}$
- The instantiation lemma is:
  - $\forall x.\ f(x) \not\simeq g(h(x)) \rightarrow f(a) \not\simeq g(h(a))$

Casting instantiation techniques: Model-based

$$E_{\mathrm{TOT}} \models \neg\psi\sigma, \text{ for some } \forall\bar{x}.\, \psi \in \mathcal{Q}$$

where E_{TOT} is a total extension of E s.t.:

- ground terms not in E necessary for evaluating Q are added
- all terms in $\mathbf{T}(E)$ not asserted equal are made disequal

▷ Consider

- $E = \{f(a) \simeq g(b),\ h(a) \simeq b\}$
- $\mathcal{Q} = \{\forall x.\, f(x) \not\simeq g(x),\ \forall xy.\, \psi\}$, $e = a$ as a default value, and

$$\begin{aligned} E_{\mathrm{TOT}} \quad &= E \cup \quad \{a \not\simeq b, a \not\simeq f(a), b \not\simeq f(a)\} \\ &\quad \cup \quad \{f(b) \simeq f(a), f(f(a)) \simeq f(a), g(a) \simeq a, g(f(a)) \simeq a\} \cup \{\ldots \end{aligned}$$

▷ Solving $\{\ldots, f(a) \simeq g(b), f(b) \simeq f(a), \ldots\} \models f(x) \simeq g(x)\sigma$ yields

- $\sigma = \{x \mapsto b\}$

▷ The lemma $\forall x.\, f(x) \not\simeq g(x) \rightarrow f(a) \not\simeq g(a)$ prevents the same E_{TOT}

How to solve the E-ground (dis)unification problem?

Entailment conditions:

- $E \models (x \simeq y)\sigma$
  - $x\sigma = y\sigma$ or
  - some t_1, t_2 s.t. $x\sigma \in [t_1]$, $y\sigma \in [t_2]$, and $[t_1] = [t_2]$

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- $E \models (x \simeq f(s_1, \ldots, s_n))\sigma$, x occurs in $f(s_1, \ldots, s_n)$,
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- $E \models (f(u_1, \ldots, u_n) \simeq g(v_1, \ldots, v_n))\sigma$ and
  - $f = g$ and $E \models u_1\sigma \simeq v_1\sigma$, $\ldots$, $E \models u_n\sigma \simeq v_n\sigma$ or
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Congruence Closure with Free Variables

Congruence Closure with Free Variables (CCFV) is a sound, complete and terminating calculus for solving E-ground (dis)unification

- ⊕ (allows for) Goal-oriented instantiation technique
- ⊕ Efficient
- ⊖ ~~Ad-hoc~~ **Versatile framework, recasting many instantiation techniques as a CCFV problem**
- ⊖ ~~Incomplete~~ **Finds all conflicting instances of a quantified formula**

Finding solutions σ for $E \models L\sigma$

$$\begin{array}{rcl} E & \models & L\sigma \\ f(a) \simeq f(c) \wedge g(b) \not\simeq h(c) & \models & (f(x) \simeq f(z) \wedge g(y) \not\simeq h(z))\,\sigma \end{array}$$

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$$x \simeq a \qquad\qquad x \simeq c$$

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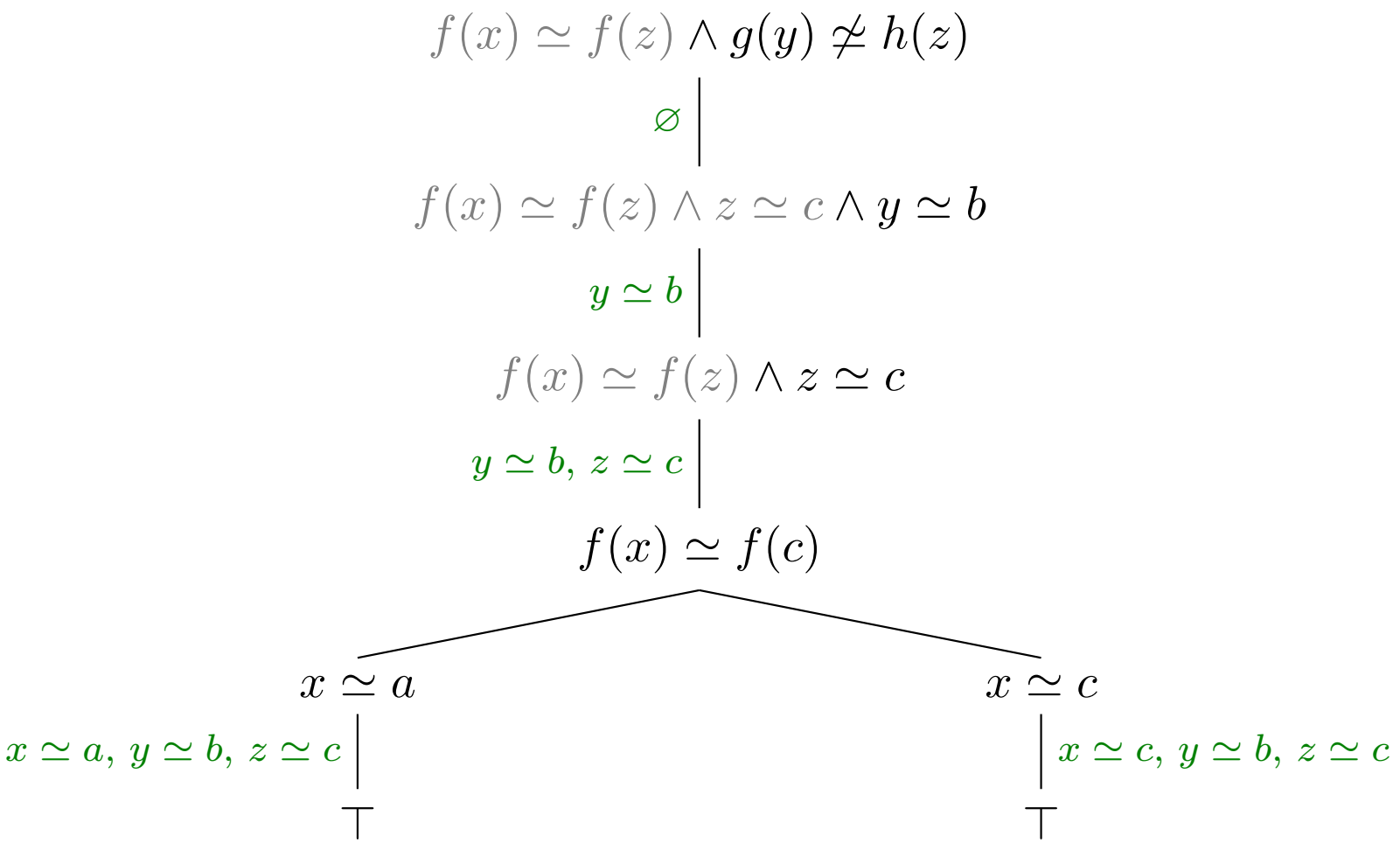
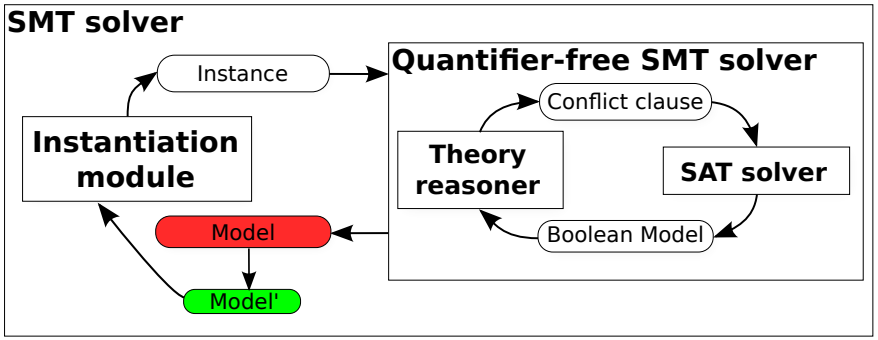


Implementation

- Model minimization
  - Relevancy
  - Prime implicant



- Top symbol indexing of E-graph from ground congruence closure

$$f \to \begin{cases} f([t_1], \ldots, [t_n]) \\ \quad \ldots \\ f([t'_1], \ldots, [t'_n]) \end{cases}$$

  - $E \models f(x)\sigma \simeq t$ only if $[t]$ contains some $f(t')$

    $E \models f(x)\sigma \simeq g(y)\sigma$ only if some $[t]$ contains some $f(t')$ and some $g(t'')$
    - Bitmasks for fast checking if symbol has applications in congruence class

- Mapping from congruence class to classes it's disequal to

Implementation

- Selection strategies

$$E \models f(x, y) \simeq h(z) \wedge x \simeq t \wedge \ldots$$

- Eagerly checking whether constraints can be discarded
  - After assigning x to t, the remaining problem is normalized

$$E \models f(t, y) \simeq h(z) \wedge \ldots$$

  - $E \models f(t, y)\sigma \simeq h(z)\sigma$ only if there is some $f(t', t'')$ s.t.

$$E \models t \simeq t'$$

Implementation

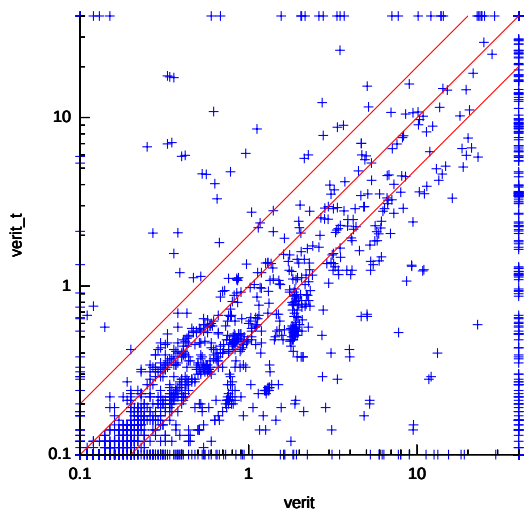
A breadth-first implementation of CCFV:

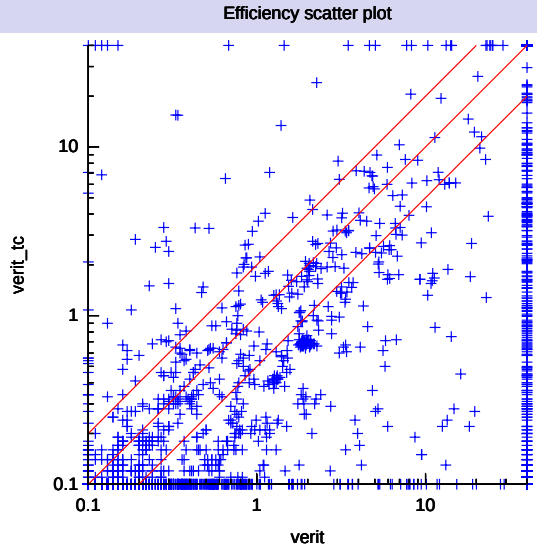
- ▷ Explores sets of solutions at a time

$$E \models \underbrace{\underbrace{\ell_1 \downarrow \mathfrak{S}_1}_{\text{individual solutions for each literal}} \sqcap \ldots \sqcap \underbrace{\ell_n \downarrow \mathfrak{S}_n}_{\text{individual solutions for each literal}}}_{\mathfrak{S}} \wedge \ldots \wedge \underbrace{\ell_1 \downarrow \mathfrak{S}_1 \sqcap \ldots \sqcap \ell_n \downarrow \mathfrak{S}_n}_{\text{combination of compatible solutions}}$$

- ⊕ Heavy use of memoization
- ⊖ Bottleneck in merging solution sets

Evaluation

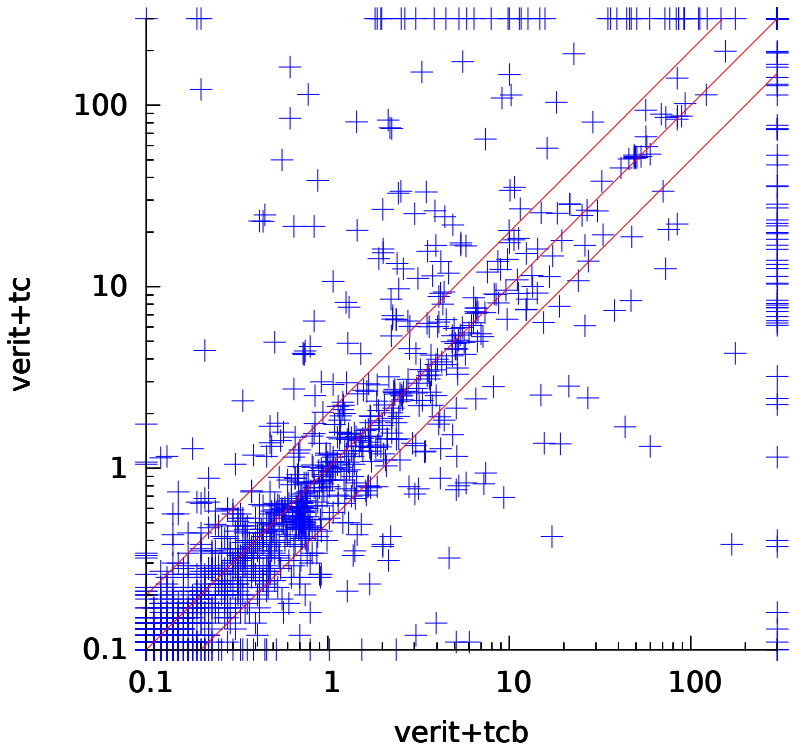




Logic	Class	Z3	CVC4	verit+tc	verit+tcb	verit+t	verit
UF	grasshopper	418	411	430	**435**	418	413
	sledgehammer	1249	**1438**	1277	1278	1134	1066
UFLIA	boogie	**852**	844	706	690	660	661
	sexpr	**26**	12	7	7	5	5
	grasshopper	341	322	326	**361**	340	335
	sledgehammer	1581	**1944**	1790	1799	1620	1569
	simplify	**831**	766	803	801	735	690
	simplify2	**2337**	2330	2307	2303	2291	2177
	Total	7635	**8067**	7676	7678	7203	6858

- ▷ experiments in the "UF", "UFLIA", "UFLRA" and "UFIDL" categories of SMT-LIB, which have $10\,495$ benchmarks annotated as *unsatisfiable*, with 30s timeout. Circa 2017.

Depth-first vs Breadth-first CCFV



The depth-first CCFV outperforms its breadth-first counterpart by a small margin.

Both perform well and are viable approaches

* experiments in the "UF", "UFLIA", "UFLRA" and "UFIDL" categories of SMT-LIB, which have $10\,495$ benchmarks annotated as *unsatisfiable*, with 100s timeout.

Limitations

- Ground congruence closure not closed to disequality entailment

  E.g. $g(f(a), h(b)) \not\simeq g(f(b), h(a)) \in E$ should lead to adding $a \not\simeq b$ to E

- No learning when backtracking

- Hard to check entailment for theories other than EUF

$$f(1) \simeq 5 \overset{?}{\models}_{\text{LIA}} (f(x+y) \leq x+2y)\sigma$$

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  Yes, for $\sigma = \{x \mapsto -3, y \mapsto 4\}$

An extension for HOL

- Build on initial extension of SMT solvers to HOL [BRO+19]
- Lifting CCFV allows directly lifting compatible instantiation techniques
- Issues
  - Currying complicates indexing
    - Terms now have the form $@(@(f, a), b)$
  - Higher-order unification (i.e. lambdas) complicates entailment checking
    - Equalities between functions
    - Unification with different arguments
    - Undecidability

Lifting CCFV to λfHOL via SAT encoding [ST20]

- Simpler fragment as sandbox
- Encode entailment checks to SAT
  - Easier to grasp
  - Free learning
  - Allows optimizations from "global reasoning"
- Build substitutions from SAT models
- Still very much in progress

Congruence Closure with Free Variables: A quantifier-instantiation framework for SMT

Haniel Barbosa

SRI Formal Topics Seminar

2020-07-16, The Internet

CCFV calculus

$$\frac{E_\sigma \Vdash_E x \simeq s \wedge C}{E_\sigma \cup \{x \simeq s\} \Vdash_E \underline{rep}(\{x \simeq s\}, C)}\ \textsc{Assign} \qquad \text{if } x \notin \mathsf{FV}(s)$$

$$\frac{E_\sigma \Vdash_E x \simeq f(\bar{u}) \wedge C}{E_\sigma \Vdash_E \bigvee_{[t] \in E^{\mathrm{CC}},\ f(\bar{t}) \in [t]} (x \simeq t \wedge u_1 \simeq t_1 \wedge \cdots \wedge u_n \simeq t_n \wedge C)}\ \textsc{Uvar} \quad \text{if } x \in \mathsf{FV}(f(\bar{u}))$$

$$\frac{E_\sigma \Vdash_E f(\bar{u}) \simeq f(\bar{s}) \wedge C}{E_\sigma \Vdash_E (u_1 \simeq s_1 \wedge \cdots \wedge u_n \simeq s_n \wedge C) \vee \bigvee_{[t] \in E^{\mathrm{CC}},\, f(\bar{t}) \in [t],\, f(\bar{t'}) \in [t]} \left(\begin{array}{l} u_1 \simeq t_1 \wedge \cdots \wedge u_n \simeq t_n \wedge \\ s_1 \simeq t'_1 \wedge \cdots \wedge s_n \simeq t'_n \wedge C \end{array} \right)}\ \textsc{Ucomp}$$

$$\frac{E_\sigma \Vdash_E f(\bar{u}) \simeq g(\bar{s}_m) \wedge C}{E_\sigma \Vdash_E \bigvee_{\substack{[t] \in E^{\mathrm{CC}}, \\ f(\bar{t}) \in [t],\, g(\bar{t'}_m) \in [t]}} \left(\begin{array}{l} u_1 \simeq t_1 \wedge \cdots \wedge u_n \simeq t_n \wedge \\ s_1 \simeq t'_1 \wedge \cdots \wedge s_m \simeq t'_m \wedge C \end{array} \right)}\ \textsc{Ugen} \quad \text{if } f \neq g$$

CCFV calculus

$$\frac{E_\sigma \Vdash_E x \not\simeq y \wedge C}{E_\sigma \Vdash_E \bigvee_{[t],\,[t']\in E^{\mathrm{CC}},\, E\models t\not\simeq t'} (x \simeq t \wedge y \simeq t' \wedge C)}\ \textsc{Dvar}$$

$$\frac{E_\sigma \Vdash_E x \not\simeq f(\bar{s}) \wedge C}{E_\sigma \Vdash_E \bigvee_{\substack{[t],\,[t']\in E^{\mathrm{CC}},\\ E\models t\not\simeq t',\, f(\bar{t'})\in[t']}} (x \simeq t \wedge s_1 \simeq t'_1 \wedge \cdots \wedge s_n \simeq t'_n \wedge C)}\ \textsc{Dfapp}$$

$$\frac{E_\sigma \Vdash_E f(\bar{u}) \not\simeq g(\bar{s}_m) \wedge C}{E_\sigma \Vdash_E \bigvee_{\substack{[t],\,[t']\in E^{\mathrm{CC}},\, E\models t\not\simeq t',\\ f(\bar{t})\in[t],\, g(\bar{t'}_m)\in[t']}} \left(\begin{array}{l} u_1 \simeq t_1 \wedge \cdots \wedge u_n \simeq t_n \ \wedge \\ s_1 \simeq t'_1 \wedge \cdots \wedge s_m \simeq t'_m \wedge C \end{array}\right)}\ \textsc{Dgen}$$

$$\frac{E_\sigma \Vdash_E C_1 \vee C_2}{E_\sigma \Vdash_E C_1 \qquad E_\sigma \Vdash_E C_2}\ \textsc{Split} \qquad \frac{E_\sigma \Vdash_E \ell \wedge C}{E_\sigma \Vdash_E C}\ \textsc{Yield} \quad \text{if } E \models \ell$$

$$\frac{E_\sigma \Vdash_E \ell \wedge C}{E_\sigma \Vdash_E \bot}\ \textsc{Fail} \quad \text{if } \ell \text{ is ground and } E \not\models \ell$$

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